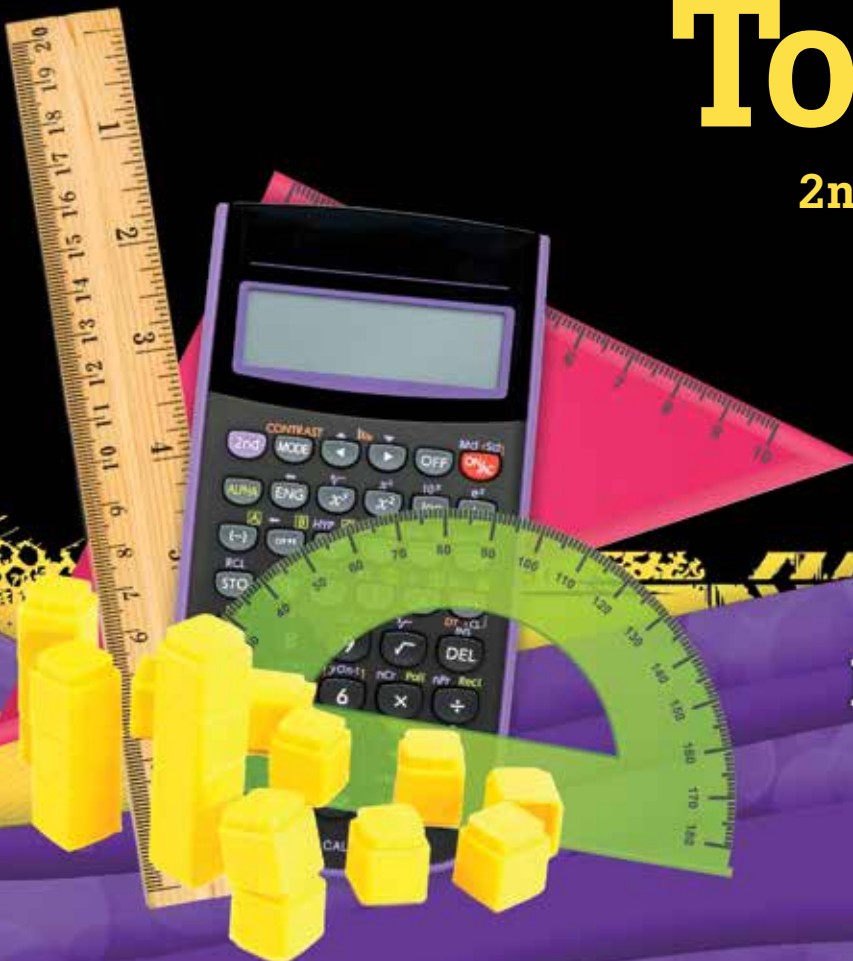




Teaching Mathematics Today

2nd Edition



Erin Lehmann

Forewords by
Sharon Rendon
Diana Wilson

Effective Teaching in
Today's Classroom



Teaching Mathematics Today

2nd Edition

Author

Erin Lehmann, Ed.S.

Forewords

Sharon Rendon, MSCI Math Specialist
Diana Wilson, Ed.D.



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Corinne Burton, M.A.Ed., *President*;
Kimberly Stockton, M.A.Ed., *Vice President of Education*; Sara Johnson, M.S.Ed., *Content Director*;
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Shell Education

5301 Oceanus Drive
Huntington Beach, CA 92649-1030
<http://www.shelleducation.com>

ISBN 978-1-4258-1207-2

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ForeWords

In *Teaching Mathematics Today, 2nd Edition*, Erin Lehmann outlines specific steps educators can take to transform their classrooms into an environment that is focused on student learning versus a culture of isolation and solely “coverage” of math content. This book is exactly what mathematics educators need to transform classrooms into a dynamic culture with higher levels of learning for everyone involved, where mathematical sense making is valued and expected of each and every student.

Erin draws from research, authors, colleagues, and her own rich experience as a mathematics teacher, coach, and professional developer to present this collection of strategies and instructional practices. She truly believes that mathematics teachers do not serve as the sole source of knowledge and the changing world in which we live requires that both students and teachers learn to learn. Through these reflective practices, both students and teachers become life-long learners and problem solvers that value understanding and are confident to tackle non-routine situations. Mathematics in a technologic world demands a new set of skills and Erin describes these skills and provides support for educators in the teaching of those skills.

I am fortunate to have worked alongside Erin on a regular basis for the past five years and have seen these strategies and practices transform classrooms and support students as they learn rigorous and challenging mathematics. This book will be a resource that will impact your mathematics instruction. You’ll hear Erin’s coaching voice nudge you to reflect on your beliefs, instruction, and practice and then take risks to implement change. As a reflective practitioner, this resource will provide guidance as you negotiate roadblocks, challenges and plan for future student learning.

I am convinced you will walk away inspired and prepared ready to dive in and make mathematics more meaningful for you and your students.

—Sharon Rendon, MSCI Math Specialist
Coaching Coordinator, CPM Educational Program

The day I met Erin Lehmann I pulled up to North Middle School where the Principal was standing outside greeting tardy students and sending them hastily into the building. I asked him where Erin's office was, she was his full-time math coach, and he said, "Have you met Erin yet? If not, be ready." I was intrigued.

Today I am still intrigued by Erin. She has now expanded her coaching role to the district level and is a proven force in building teachers' instructional capacity in an inquiry-based math classroom. In *Teaching Mathematics Today* Erin offers the next best thing to her being right there beside the teacher. In a concise and comprehensive A-Z, she gives rich explanations and concrete practices that jump from the pages and into action in our classrooms at Rapid City Area Schools.

Teaching mathematics today requires new skill sets which are drastically different than the traditional practices many teachers experienced as students themselves. University mathematics professors struggle to prepare today's math educators for the same reason. Traditional methods teach students to use algorithms to get the answers the teachers want. Today's mathematics instruction must help students gain a deep conceptual understanding of how and why math works and how to apply math in ways that make sense to them.

While many teachers are willing to transform their traditional mathematics instruction to an inquiry-based model, many only reach a superficial level of proficiency. Temptation to continue past practice weighs strong on teachers who find it difficult to let students struggle and persevere in solving problems. They may find it difficult to ask the right questions or may not have the right learning environment to facilitate students' mathematical discourse. If proficiency with inquiry-based instruction is the goal, Erin offers support and constructive examples that will build teachers confidence and competence.

Whether you coach mathematics teachers, whether you supervise mathematics teachers, or whether you are the mathematics teacher, *Teaching Mathematics Today* is a must read.

—Diana Wilson, Ed.D., Director
Office of Curriculum, Instruction and Assessment
Rapid City Area Schools

Introduction

Gone are the days of stand-and-deliver lectures, with the teacher hoping that the students will remember what was said to them—or better yet, that through those lectures students will be able to internalize the complex skills associated with mathematical problem solving. Today’s mathematics teachers need to develop trusting relationships with students and create a culture in which students take risks, explore the intricate aspects of problem solving, and participate in mathematical dialogue daily. The classroom is now a place where students can collaborate, explain, question, and justify mathematics.

Teachers should not undertake this shift in practice alone. Being an educator is beyond demanding, and a support system within a school is one way to ease the pressure of being a teacher. One way of easing this pressure is through the use of professional learning communities (PLCs). A professional learning community is a systematic way to help teachers meet the learning needs of all students. In a PLC, teachers work together to impact their classroom practices in ways that will lead to better results for their students, for their team, and for their school (DuFour et al. 2010). It is beneficial for teachers to collaborate with each other when studying state and district standards, creating units, planning summative and formative assessments, analyzing student work and data, and implementing differentiation. Teachers truly are smarter when they focus on the right work together. It takes commitment by teachers and the willingness to learn and study together to build this shared knowledge.

Students need both procedural and conceptual knowledge in order to learn and understand mathematics (NCTM 2000). Knowledge of the procedures and formulas is critical to overall proficiency in mathematics. Also, exploration of the concepts through concrete experiments and manual manipulation is vital to students’ overall understanding of the “why” in mathematics instruction. Emphasizing a high degree of procedural proficiency without developing conceptual knowledge is ineffective, and

focusing only on conceptual knowledge will not help students achieve in the classroom and in real-world situations. It is necessary to provide focused instruction that moves students from the concrete to the abstract, and then to the application of the concept (Marzano 2003; Sutton and Krueger 2002).

While no single method of instruction has been proven to be the best way to teach mathematics, using research-based designs and procedures has helped educators recognize the best ways to approach learning goals in mathematics (Hiebert and Grouws 2007). Today's mathematics teacher needs to develop a scope and sequence that integrates multiple opportunities for practice prior to an assessment, and time to reteach and revisit those skills that students have not mastered after the assessment. When students are given sufficient practice, they can attempt to use a newly-learned skill in various situations with accuracy so that the skill will be retained (Sousa 2006).

About This Book

Teaching Mathematics Today offers research-based explanations of the strategies that are most critical and highly effective when teaching mathematical concepts, and it promotes the use of best practices. This book reinforces the role of the teacher, who is actively engaged with others, to help students become successful mathematicians. It also offers practical explanations for how to unpack state and national standards to find what students should know, understand, and be able to do.

Teaching Mathematics Today is intended as a guide for mathematics teachers. The book is designed to address grade levels K–12, and it can also be adapted for the various disciplines of mathematics. Here are a few ways educators can use this book:

- Teachers can read the extensive explanations of the mathematics teaching strategies and integrate them into their own lessons.
- Teachers might use this book to improve the effectiveness of mathematics instruction in their classroom.
- A school, mathematics department, or PLC might work through the entire book when streamlining and improving a mathematics program.
- Mentors and coaches can use this book as a guide as they work with teachers, and they can read this book in a study group to discuss best practices in mathematics teaching.
- New and veteran teachers can read and apply the techniques described in the chapters to their current instruction of mathematical concepts.
- Teachers can deepen their knowledge of differentiation to meet the different needs of, and offer access to core curriculum to struggling students and English language learners.

Ultimately, the suggestions in this book will assist teachers in developing their toolbox of research-based teaching strategies to address different learning styles, engage students, and differentiate instruction.

How This Book Is Organized

This book is designed to be read in sequential order. Each chapter focuses on a different aspect of teaching in a mathematics classroom. Here's a brief summary of the topics covered:

- **Chapter 1—Understanding the Role of the Teacher in the Mathematics Classroom:** It is imperative for teachers to develop character trust and competency trust with their students in order to facilitate learning. Students need to wrestle with mathematical concepts in a “productive struggle” and have opportunities to explain and justify their thinking. This chapter provides multiple questioning strategies teachers can use immediately. Because teachers can no longer afford

to work in isolation, this chapter also covers collaborative teams of teachers whose members work interdependently to achieve common goals. PLC members are mutually accountable and realize that all of their efforts must be assessed on the basis of results rather than intentions (DuFour et al. 2010).

- **Chapter 2—Managing a Successful Mathematics Classroom:** When strong classroom management techniques are implemented, students have more opportunities for learning. It is important for teachers to have routines and procedures and review them frequently with students. This chapter provides multiple resources for mathematics teachers as they plan for and develop their own classroom management strategies. The planning tools in this chapter will help guide teachers as they implement a workshop model, allowing students to explore and be engaged in mathematics and establishing a sense of community.
- **Chapter 3—Planning for Instruction:** When planning for instruction, it is crucial for mathematics teachers to unpack standards for their grade level and course. Teachers need to attend to the rigor at which the standard is written and hold students to that level of understanding. This chapter offers resources to use when unpacking standards and writing learning targets for instruction.
- **Chapter 4—Implementing Mathematical Practice and Process Standards:** Mathematical practice standards are the “how” in terms of content proficiency. Teachers need to provide opportunities for students to practice engaging in mathematical dialogue in which they defend, explain, justify, and question each other. This chapter provides tools and strategies to reinforce mathematical practices and incorporate them into the mathematics classroom through cooperative learning. When students are actively motivated and busy reaching learning goals, they are also constructing knowledge and moving toward successful mastery of the mathematical content standards.
- **Chapter 5—Building Conceptual Understanding:** Research shows that students who are not successfully mastering mathematical concepts tend to demonstrate slow or inaccurate retrieval of basic mathematical facts, lean toward impulsivity when solving problems, and have difficulty forming mental representations of mathematical concepts or keeping information in working memory (Gersten and Clarke 2007). This chapter offers a step-by-step process to teach students problem-solving strategies along with various vocabulary

development activities. The set activity examples are meant to develop independent, competent student problem-solvers.

- **Chapter 6—Assessing Students:** To reach the goal of all students mastering the given curriculum with appropriate instruction, materials, and support, mathematics teachers must use summative and formative assessment strategies “minute by minute and day by day, to adjust their instruction to meet their students’ learning needs” (Wiliam 2007). Assessments provide teachers with the necessary data to understand which students are struggling in specific areas of the curriculum and which students need enrichment. This chapter provides strategies, charts, and rubrics for summative and formative assessments, as well as ways to use data to further drive instruction in the classroom.
- **Chapter 7—Supporting Instruction Through Differentiation:** Students should have multiple experiences with topics, allowing them to integrate the topics into their knowledge base (Marzano 2003). However, not all students process the new information in the same way or bring the same skill set to the learning experience. Some students need extra time to process concepts and look at problems in different ways (Sutton and Krueger 2002). Other students may require intense intervention—further teaching or material presented in multiple ways. This chapter provides charts, strategies, and tips for identifying individual student needs and ways to differentiate mathematics instruction to meet those needs.
- **Chapter 8—Integrating Mathematics Across the Curriculum:** Rather than working on subjects in isolation from one another—studying reading apart from writing and apart from math, science, social studies, and other curricular areas—children learn best when they are engaged in inquiries that involve using language to learn and that naturally incorporate content from a variety of subject areas (NCTE 1993). It is important for students to understand that education is not a series of compartmentalized subjects that have nothing to do with one another. Rather, students need to realize that learning is more like a tapestry, where all subjects are woven together to create a broad scope of understanding that is ultimately most useful when all the strands fit together. This chapter provides suggestions and strategies for teachers to integrate mathematics across the curriculum. It includes information about reaching all learners and broadening students’ understanding of mathematical concepts.

- **Appendix A:** Teaching mathematics in today's diverse classrooms can be challenging, but it also provides teachers with many exciting opportunities to pass on life skills as well as mathematical knowledge. This section offers general information and suggestions for all teachers and provides additional tips and advice for new teachers.

Conclusion

This book is intended to help teachers navigate today's mathematics classroom. Closed doors are a thing of the past; conversations with fellow educators will help teachers meet the diverse needs of all learners. With a focus on student thinking and learning, this book provides strategies and techniques for teachers to explore and try. Change can only happen through effective teaching by first understanding the role of the teacher, as described next.

Chapter 1

Understanding the Role of the Teacher in the Mathematics Classroom



“Teachers’ attitudes and expectations, as well as their knowledge of how to incorporate the cultures, experiences, and needs of their students into their teaching, significantly influence what students learn and the quality of their learning opportunities.”

(Banks et al. 2005)

The diversity in today’s mathematics classroom requires agility on the part of teachers. Teachers need to not only master mathematical teaching, but they must also demonstrate an innate understanding of the different life experiences students bring to school. According to Megan Tschannen-Moran (2004, 12), “Students who do not trust their teachers or each other will be likely to divert energy into self-protection and away from engagement with the learning task. Moreover, students who do not feel trusted by their teachers and administrators may create barriers to learning as they distance themselves from schools and build an alienated, rebellious youth.”

This chapter aims to deepen teachers’ understanding of trust and how the different types of trust help teachers navigate the diversity in a classroom. As important as trust is in the classroom, so is an understanding of growth and fixed mind-sets. This chapter outlines how mind-set influences the culture of learning within a classroom.

Student-constructed learning reinforces that learning is an active, meaning-making process, and this process takes time. The strategies listed in this chapter will help teachers provide opportunities for students to do the talking, which leads to understanding. Because planning for success in isolation is challenging, teachers may consider participating in a professional learning community (PLC), a working team of teachers focusing on student learning and increasing the probability of success.

Creating Trusting Relationships with Students

First impressions are critical in developing relationships with students; a student's initial encounter with a teacher can set the tone for his or her engagement and growth the rest of the school year. A teacher should have a warm and welcoming demeanor that invites students to be excited about the adventures they are about to experience in the mathematics classroom.

Students are more willing to take risks and be vulnerable if they trust their teacher and believe that the classroom environment is safe and nurturing. Listening to students is one of the first ways a teacher can begin to build trust. In many classrooms, teachers tend to do all of the talking and they take little time to truly listen to their students. Saying "Tell me more about that" is an effective way to get kids talking. Also, before school, during lunch, in the halls, and at sporting events are all good times when teachers can ask students personal questions and take note of their responses. Students are fairly in tune with body language, so teachers should give full attention to the speaker. True listening is an honest act and not a phony gesture.

Classroom teachers who model both strong character and strong competence are positioned to develop high-trust cultures or classrooms (Moore 2009). Character trust and competency trust, and the ways teachers can develop them, are explained in more detail in the sections that follow.

Character Trust

Character trust includes the behaviors within oneself and needs to be developed and demonstrated each and every day. Students tend to respect the teachers who truly care about them, and they often speak of these teachers highly. Robert and Jana Marzano (2003, 41) state, "If a teacher has a good relationship with students, then students more readily accept the rules

and procedures and the disciplinary actions that follow their violations.” Without realizing it, educators will also do a lot of teaching outside of their academic content. Teachers are often role models children learn from, so educators have a valuable opportunity to teach positive character traits alongside mathematics. Sometimes this is as simple as greeting students as they walk in the door. Other character traits to model are as follows:

- Truthfulness
- Dependability
- Reliability
- Flexibility
- Patience
- Tolerance
- Availability
- Responsibility

Competency Trust

Competency trust is how a teacher is viewed at doing his or her job. In the eyes of students, this may include teacher preparation, content knowledge, classroom management, and communication with students. If competency trust is high, students are going to trust the teacher, and learning will be the end result. Because the mathematics teacher has created a culture of trust and acceptance, it is okay for students to not know and understand a concept. This high-tolerance classroom, where mathematical conversations allow for questioning and mistakes lead to deeper learning, permits students to be more vulnerable. Teachers strive to get students to talk and collaborate, and they can model the types of positive mathematical conversations they want to hear. An effective way to initiate a mathematical conversation is to ask for a volunteer; students are often eager to do this. Setting the stage for mathematics talk is a way to reinforce this positive behavior again and again. It is important to note when students are having good mathematical conversations. Write down student phrases or peer words of encouragement and share these at the end of class. “How did you get that answer?” will lead to students viewing themselves as mathematicians. This type of positive reinforcement is the structure of a constructive and safe mathematics classroom. If competency trust is low, it is likely there will be management issues, and learning will be secondary to everything else.

Who else should be more excited about mathematics than a math teacher? The enthusiasm for the content needs to be contagious! Each and every day is a new opportunity to express this. Students will be more attentive if the teacher truly loves what he or she is doing and expresses joy while teaching.

Not many people can get excited about the Pythagorean theorem—a math teacher is usually the only exception. Not only is the content exciting, but observing learning is just as exciting. When a student exclaims, “Oh, I get it!” while the teacher walks around the classroom, that teacher should be sure to smile, give a high-five, and celebrate. These are the moments that students will remember down the road.

Mathematics is a series of stories, and when a teacher can relate concepts to students’ lives during class, that teacher is making real-life connections that will be memorable for students. The feeling of importance or being noticed is something students look forward to. For example, a word problem could contain the names of students in the class or involve activities students participate in. Perhaps there are fundraisers the class is participating in or mathematics questions students think of that the teacher can use as part of everyday lessons including students.

Finally, the most important thing a teacher can do to create a trusting classroom is to believe in students. “All students can learn math” should be a mantra the teacher repeats daily. Time is the variable in this equation and learning is the constant. Students come to mathematics class at all different levels of readiness. Teachers need to believe *all* students can and will learn mathematics in the classroom each and every day.

Developing a Culture of Learning

The teacher sets the tone for the culture of the classroom. If the teacher is sitting behind a desk as students walk in the room, he or she may not be fostering a welcoming environment. However, if the teacher is up and greeting students enthusiastically, the ambiance is convivial and inviting. The moment students enter the classroom, it is the teacher’s responsibility to help them see their greatest potential in a safe and nurturing atmosphere.

In life, students will encounter successes and failures, especially in a mathematics classroom. Intelligence is not fixed, but many students think they are bad at mathematics because it does not come easily to them. Hard work, motivation, and the belief that students can learn can make the difference in learning. A teacher needs to set the stage so that every student can and will learn. Every child has potential, and the teacher should focus on and praise the efforts, risks, and challenges students are undertaking in

terms of learning, not necessarily on fixed student intelligence. It is hard to break the stereotype of intelligence being fixed, but when educators promote the fact that learning is fluid, students begin to understand that learning can happen at any given time. In a culture of acceptance and learning, students will have their “ah-ha” moments and educators will celebrate these moments and encourage more risk-taking in learning.

The sections that follow, discuss the importance of student motivation and development of a growth mind-set, praise, responsible actions, expectations, and teacher behaviors as part of the culture of learning.

Student Motivation and Mind-Set

Low student motivation is an issue many educators have observed over the years in the educational system. Fear and intimidation are unfortunately used in many schools today and create a natural resistance in most students. Despite compelling evidence to the contrary, some teachers still believe that fear—fear of failure, fear of an unwanted call home, fear of the teacher, fear of ridicule, or fear of an unpleasant consequence—is a prime motivator for students to do high-quality work (Sullo 2009). In this type of fear-based environment, students will try to be invisible and avoid any and all chance of being noticed, even if this means not asking questions because they do not understand. When fear is instilled in people, adrenalin (the hormone released into the body when we experience fear) inhibits the functioning of the frontal cortex, which is the problem solving part of the brain. When a teacher says, “You really don’t know how to add those two fractions. What grade are you in?” the student’s thinking stops. When student motivation decreases, it becomes hard for students to think, problem solve, or make good decisions. In the end, using fear and intimidation does nothing to enhance learning.

“How can we motivate students?” is a question educators have been asking for years. Motivation should be an internal drive in everyone, but it seems to be missing in some students. Dangling a carrot or reward in front of students to succeed will work for some, but not for others. As mentioned previously, for years teachers tried using fear as a motivating factor, but that tactic will never achieve internal motivation. According to Bob Sullo (2009), “When students find school and learning to be a need-satisfying experience, they will put working hard and learning into

their internal world and will be the academically motivated students we would like them to be.” One way to help students experience school as satisfying is to grow their mind-set. If students believe that learning is based more on effort than on intelligence, they will understand the brain is malleable and not fixed.

Carol Dweck’s book *Mindset* explains the difference between a fixed and a growth mind-set. Dweck and her colleagues conducted an experiment where they followed several hundred students in New York City during their seventh-grade transition. They measured the students’ mind-sets at the beginning of the school year and monitored their grades over the next two years to see how they coped with new challenges. Despite their differing mind-sets, students entered seventh grade with similar mathematical achievement, but their grades split apart in their first term and continued to diverge over the next two years. The students with the growth mind-set (those who believed that intelligence could be developed) significantly outperformed those classmates with a fixed mind-set (Dweck 2008).

A *fixed mind-set* is when looking naturally smart is more important than trying and putting forth effort. A student with a fixed mind-set might see effort as a weakness, and if learning does not happen easily and quickly, he or she gives up. Phrases associated with fixed mind-set might include the following:

- “I can’t.”
- “This is too hard!”
- “I give up!”
- “This is so stupid.”
- “I don’t understand decimals, fractions, and percents.”

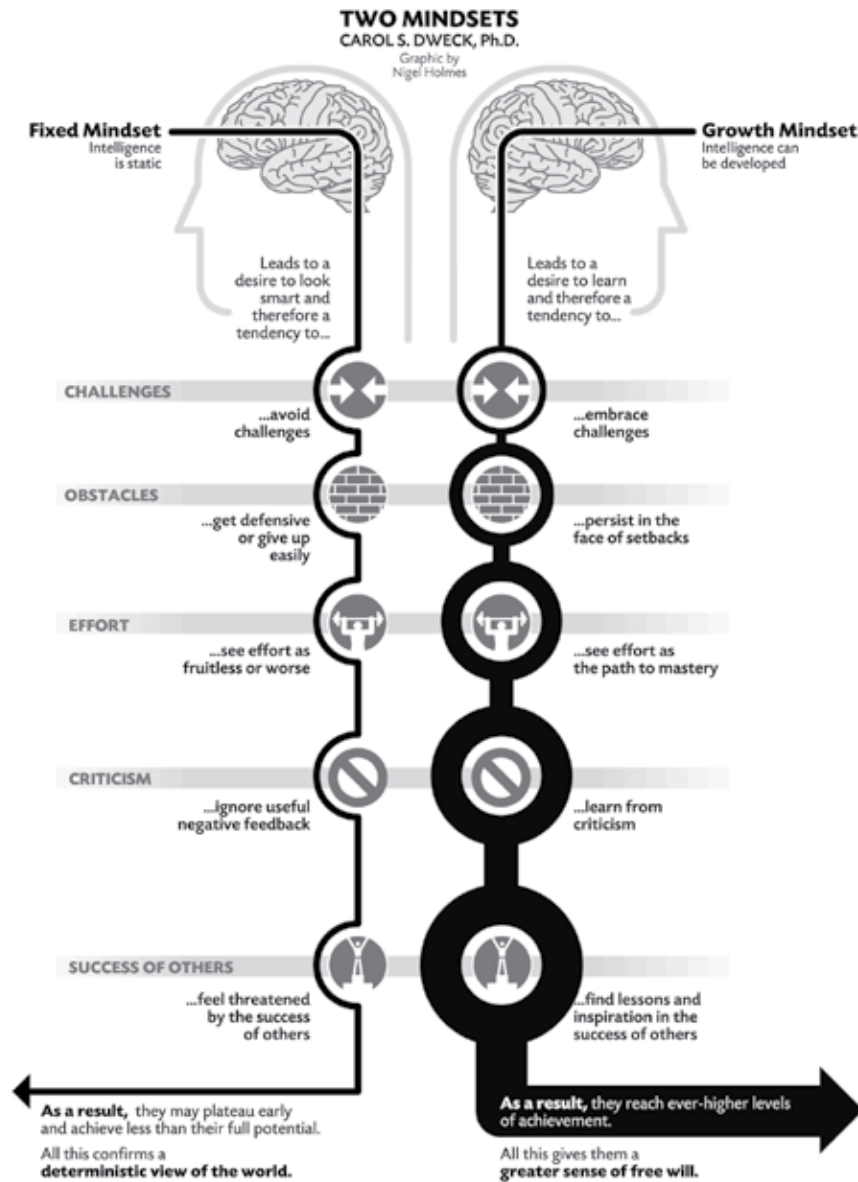
A *growth mind-set* is when a student believes that intelligence is malleable and can increase through hard work and effort. Mistakes happen, and students with a growth mind-set understand that this is when true learning occurs. Students with a growth mind-set look forward to challenges and persevere when confronted with obstacles. Phrases associated with a growth mind-set might include the following:

- “This is hard; I can’t wait to figure it out!”
- “Don’t stop me now—I want to keep working.”
- “I can do this!”
- “Don’t give me a hint. I know I can find the solution.”
- “I don’t understand decimals, fractions, and percents, yet.”

To develop students’ mind-sets, a teacher could have the students write and reflect on how they feel as learners. The teacher can put phrases like the ones previously mentioned on the board and have students note which ones they tend to say more often. If students lean toward the fixed mind-set, the teacher can remind them that the brain is malleable and it is a choice if they want to have a growth or fixed mind-set. The students can rate themselves daily from a 1 (having a fixed mind-set) to a 5 (having a growth mind-set) and then reflect on why they chose the rating they did. The teacher can ask students to write one goal that they will work on immediately to grow their mind-set. The teacher can also ask each student what they can do to help support these goals.

Another way to grow students’ mind-sets is to have them consider Nigel Holmes’s graphic on mind-set (see Figure 1.1) and think about which side they resonate with more. The teacher can ask students to give examples of what each mind-set looks like. There are YouTube videos on growth and fixed mind-sets that the teacher can show in class and then ask students to share their thoughts and feelings. Videos for younger students include Tigger and Eeyore from *Winnie the Pooh* thinking about and working through problems. Students can observe Eeyore’s fixed mind-set and Tigger’s growth mind-set, and then the teacher can ask students what evidence supports these mind-sets. Videos for older students can include famous people who could have gotten stuck in the fixed mind-set due to comments from others but chose to ignore that feedback and continue living their dreams. Also, the teacher can have students share when they have demonstrated a growth or fixed mind-set. It can be enlightening for students to think about the choices they make each day.

Figure 1.1 Fixed vs. Growth Mind-set



(Reprinted by permission of Nigel Holmes)

Praise

Teachers should be cautious when giving praise to students. Giving praise based solely on intelligence tends to encourage a fixed mind-set. Students may start to take on less challenging activities because the harder activity might be more difficult to figure out and they would in turn appear less intelligent. Instead, praise the processes students choose. For example, if a student is working on finding the area and perimeter of a rectangle and he or she starts by using manipulatives, praise the student on the exploration and perseverance rather than focusing on the right answer. Other areas of praise might include effort, concentration, and the use of different strategies. Praising children's intelligence harms their motivation and their performance (Dweck 2008).

Responsible Actions

Responsible actions can and should be taught in a classroom. Because district and state standards likely include constructing viable arguments and critiquing the reasoning of others, mathematical conversations need to be happening regularly. To maintain a safe culture of learning and conversing, teachers need to be explicit about what responsible actions look like and sound like in a mathematics classroom. One way to address responsible actions is for teachers to brainstorm a personal list of actions that are important to them and to ask students to brainstorm a list they believe are important. For example, a teacher might value promptness, personal accountability, respect, and courage as actions he or she expects to see on a daily basis. Students might value responsibility, effort, and honesty. The teacher can then merge the two sets of beliefs so there is a shared understanding of student and teacher expectations. Once the class has one list, the teacher can create a poster of the behaviors and point out whenever a student exhibits one from the list—for example, “Connie, when you showed Emily how to change the fraction to a decimal, you did it respectfully. That helps us create a community of learners. Thank you.” The teacher can give students multiple opportunities to read and reflect on these beliefs, and when the teacher asks students to read and reflect, he or she takes time to do the same.

Expectations

Uncertainty is a scary feeling for many students. As students enter the classroom, they should know immediately what they are going to learn and how it will be measured. To honor a culture of learning, students need to know what is expected of them. Learning targets should be posted so students know what they are going to learn during the day. Each minute is vital in a mathematics classroom. By having the learning target posted, students will not waste time asking, “What are we doing today?” The criteria for success should be listed near the learning target as well. Students will know exactly what they need to do in order to be successful. (Chapter 3 focuses further on learning targets and criteria for success.)

Teaching Behaviors

The actions of the teacher impact the classroom environment. A teacher must cooperatively create an atmosphere of learning for all students. Effective management is a partnership: it depends on collaboration (Routman 2008). Positive actions by the teacher will foster effective teacher-student relationships. In a mathematics classroom, there will be high expectations for deep thinking. Students will be making mathematical mistakes due to the high rigor, which is really a good thing. According to Cathy Seeley (2014, 58), “By creating a classroom environment where mistakes are part of the learning process, we can help students revisit their errors, discuss their thinking, interact with their peers or the teacher, and approach the problem or exercise in a more productive way.”

Figure 1.2 shows how teacher actions can be viewed as positive and inviting, so students are excited to talk about mathematics; or negative, so students shut down and are turned off by mathematics.

Figure 1.2 Teacher Actions: Negative and Positive

Negative Actions	Positive Actions
Demands obedience, uses rewards and punishments as motivators	Offers choice and reassurance in a welcoming environment
Uses guilt or shames students into doing work and often criticizes students	Supports independence and expects students to contribute
Does not trust students' ability to learn	Encourages mutual respect and trusts students
Pushes for unattainable goals	Demonstrates patience and encouragement, sets attainable goals

The Teacher as Facilitator vs. Lecturer

Lately, many college programs are emphasizing the role of a teacher as facilitator. Preservice teachers are now taught not to stand in the front of the classroom and talk the entire time, but to let students discover and learn for themselves. Today's teachers encourage collaborative group work. In groups, students tend to listen and share ideas and methods. They need time to wrestle with concepts and discuss challenging problems. New teachers, whose belief is to let students construct meaning, listen and question, pause and paraphrase, will be ready to create this type of learning environment. However, when the going gets tough, new teachers often resort to teaching like they were taught: stand and deliver and put students back into rows. They abandon all student-centered learning and seek silence for a more structured traditional atmosphere. This is likely to happen and it is okay, but teachers should try again the next day. It takes time and patience, but when teachers see students putting their heads together to discuss strategies to solve a logic problem, they realize it is worth it. Students learn to think like mathematicians through collaboration and discovery, and neither is done silently.

Constructivism is a theory of learning based on the principle that learners construct meaning from what they experience; thus, learning is an active, meaning-making process (Glatthorn and Jailall 2009). It takes time and effort to plan for this type of learning, where students make discoveries

with real-world problems, but it is essential for all learners. In considering the nature of learning, constructivism involves nine basic principles (Brooks and Brooks 1999; Cornelius-White and Harbaugh 2010; Marzano et al. 1997; Larochelle, Bednarz, and Garrison 1998):

1. **Learning is not a passive, receptive process but is instead an active, meaning-making process.** It involves performing complex tasks that require active use and application of knowledge in solving meaningful problems.
2. **Learning at its best involves conceptual change—modifying one’s previous understanding of concepts so that they are more complex and more valid.** Typically the learner begins with a basic or inaccurate concept, and the learning process develops a deeper or truer understanding of the concept.
3. **Learning is subjective and personal.** Learning involves internalizing what is being learned, representing it through learner-generated symbols, metaphors, images, graphics, language, and models.
4. **Learning is situated or contextualized.** Learners carry out tasks and solve meaningful, real-world problems. Rather than doing “exercises” out of context, learners learn to solve contextualized problems.
5. **Learning is social.** Learning at its best develops from interaction with others as perceptions are shared, information is exchanged, and problems are solved collaboratively.
6. **Learning is affective.** Learning is influenced by the following affective elements: self-awareness and beliefs about one’s abilities, clarity and strength of learning goals, personal expectations, general state of mind, and motivation to learn.
7. **The nature of the learning task is crucial.** The best learning tasks are characterized by these features: optimal difficulty in relation to the learner’s development, relevancy to the learner’s needs, authenticity with respect to the real world, and challenge and novelty for the learner.